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Natural convection heat transfer from inclined isothermal plates

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Abstract—Experimental local and average heat transfer data are obtained for natural convection heat transfer from isothermal vertical and inclined plates facing upwards to air. The employed apparatus enabled the isolation of the effect of side edges, thus ensuring the condition of infinite plate width. The experiments covered both the laminar and the turbulent regions. Equations representing the results are suggested.

INTRODUCTION

MANY INVESTIGATIONS are available on natural convection heat transfer from vertical plates. Only a limited number of investigations are, however, available on inclined plates and in only some of them was the investigation extended into the turbulent region. The plates used in most cases were of 'finite width' and the results suffered from the presence of side-edge effects.

The present research work represents the results of experiments carried out to investigate local and average natural convection heat transfer from isothermal vertical and inclined plates facing upwards to air in both the laminar and the turbulent regions. The apparatus used enabled side-edge effects to be isolated thus ensuring the 'infinite width' or the very wide plate condition.

PREVIOUS WORK

A summary of previous data on inclined plates only is given hereinafter. The plate dimensions are given as length × width.

Rich [1] carried out experiments on an isothermal 400 × 100 mm plate heated in air in the laminar region in the range of θ up to 40° and $Gr_{loc}Pr$ from 10^6 to 10^9.

Kierkus [2] studied local laminar free convection from an isothermal plate in air theoretically and experimentally. The experimental plate had dimensions of 250 × 250 mm and only two angles of inclination ($\theta = 0°$ and 45°) are compared with the predicted solution.

Hassan and Mohamad [3] measured the local heat transfer from a 504 × 200 mm isothermal plate to air. The angle of inclination varied from −90° to 90° and the range of $Gr_{loc}Pr$ was from 300 to 3×10^8.

Vliet [4] measured the local heat transfer from a constant flux heated plate (121.9 × 91.44 mm) to water and air at θ from 0° to 60°.

Fujii and Imura [5] used two plates 300 × 150 mm and 50 × 100 mm in the range of θ from −90° to 90°. The average heat transfer was measured but the heating condition was neither isothermal nor constant flux.

Black and Norris [6] carried out an interferometrical study on an isothermal plate to air in the range of θ from 0° to 80° and $Gr_{loc}Pr$ from 4×10^5 to 9×10^9.

Lloyd and Sparrow [7] investigated the transition from the laminar to the turbulent flow on an isothermal plate (152 × 103 mm) using an electro-chemical method in the range of θ from 0° to 60°.

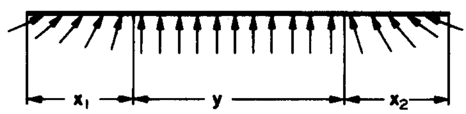

FIG. 1.

APPARATUS

The general shape of the pattern of flow of natural convection currents to a hot vertical plate projected on a horizontal plane normal to the surface of the plate is shown in Fig. 1. Whereas the flow to the central zone of the plate (marked Y) is two-dimensional, there exist two side zones (marked x_1 and x_2) to which the flow is three-dimensional. The heat transfer in the side zones must, therefore, be higher than in the central zone.

In order to determine the heat transfer from a plate of infinite width (a very wide plate) the side zones have to be isolated and the apparatus employed must enable the heat transfer from the central zone to be determined separately.

The apparatus used is shown diagrammatically in Fig. 2. It consists of a polished brass plate (1) 1300 mm long, 650 mm wide and 2 mm thick. The back of this test plate, which was heated by steam condensing at atmospheric pressure, was subdivided by longi-

NOMENCLATURE

b	distance from plate side edge
Gr	Grashof number
h	heat transfer coefficient
L	plate length
Nu	Nusselt number
q	rate of heat transfer from each compartment
Pr	Prandtl number
t	temperature
x	distance from lower edge of the test plate.

Greek symbols

θ	angle of plate inclination from the vertical position.

Subscripts

a	ambient
av	average
c_1, c_2	critical (beginning and end of transition)
f	film
in	infinite
loc	local
s	surface.

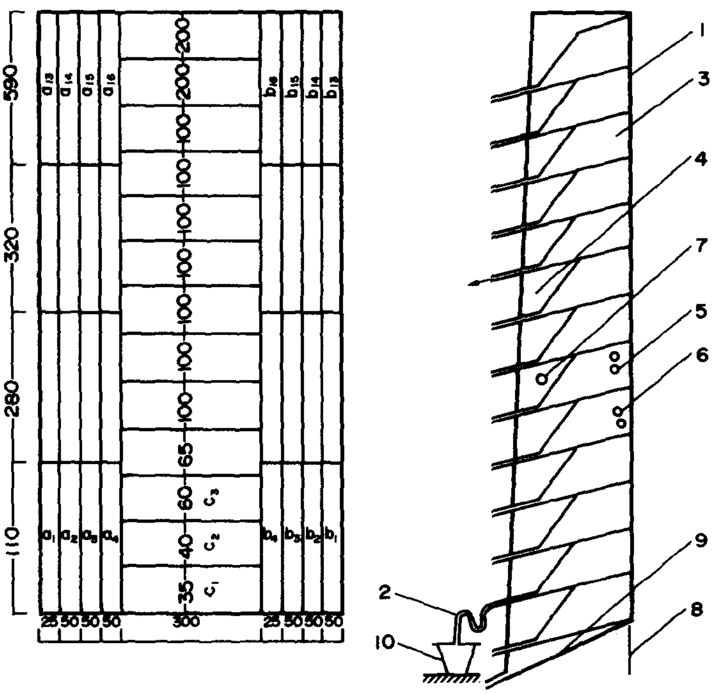

FIG. 2.

tudinal partitions into one central zone and two side zones. Each of the two side zones was 175 mm wide and the central zone was 300 mm wide. By means of transverse partitions the central zone was subdivided into 13 compartments $c_1, c_2, \ldots, c_{13}$. Each of the two side zones was subdivided into 16 compartments $a_1, a_2, \ldots, a_{16}$ and $b_1, b_2, \ldots, b_{16}$. The dimensions of these central and side compartments are shown. Arrangements were made for the condensate to be collected separately from each compartment through syphon traps (2). The back and the sides of the compartments were insulated by steam condensing in an outer jacket (4). Steam entered the compartments slightly superheated through tubes (5). The amount of superheat was about 8°C. This ensured that the steam entered perfectly dry and that the condensate in each compartment resulted from the heat lost through the corresponding section of the plate. The error due to this amount of superheat was some 0.5% of the total heat transfer. Vent tubes (6) ensured atmospheric pressure in the compartments. The apparatus was mounted on a pivot (7) so that the test plate could be tilted around a horizontal axis at any angle. A wooden flat leading edge (8) formed an extension to the plate surface and the surface of the outer steam jacket was insulated by a wooden cover (9).

As the heat transfer coefficient on the steam side surface of the plate is considerably higher than on the air side surface, the temperature of the former was taken equal to the saturated temperature of the steam at the pressure registered by the barometer at the time of the experiment. The air-side plate temperature was practically the same as the steam-side temperature as the temperature drop in the metal due to conduction was negligible (about 0.01°C).

The heated test plate was polished and an emissivity equal to 0.08 [8] was taken to calculate the heat lost by radiation. The radiation loss was about 8% of the total heat transfer. A 20% error in the assumed emissivity value would result in less than 2% error in the calculated convective heat transfer coefficient.

RESULTS

The experiments were carried out with the test plate tilted at six inclination angles from the vertical ($\theta = 0°$, $15°$, $30°$, $45°$, $60°$ and $80°$). The physical properties were taken at the mean film temperature, $t_f = (t_s + t_a)/2$.

EFFECTIVENESS OF SIDE COMPARTMENTS

It was necessary to make sure that the side compartments could isolate side-edge effects effectively. The widthwise variation of the rate of heat flux q from the side compartments (as calculated from the steam condensed in the compartments) is plotted against plate width b in Fig. 3 for the vertical position. The arrangement of the compartments enabled seven points on the q–b curve to be obtained for four compartment heights (0–110, 110–390, 390–710 and 710–1300 mm). It can be seen that q is a maximum at the plate side edge. As the distance from the side edge increases q decreases until it becomes practically constant at a distance E equal to about 135 mm. For other angles of inclination E was less than 135 mm. As the central zone lies at a value of $b = 175$ mm this zone was always far from side-edge effects and therefore represents a plate of infinite width.

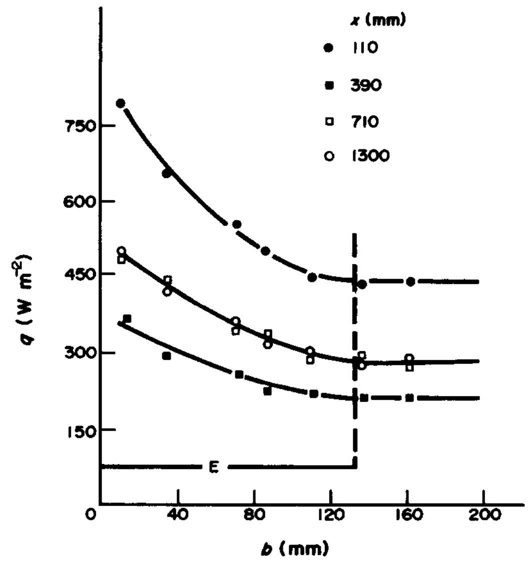

FIG. 3.

LOCAL HEAT TRANSFER

Variation of local heat transfer coefficient with plate length

The variation of the local heat transfer coefficient h_{loc} as calculated from the central compartments, with plate length x is shown in Fig. 4 for the angles of inclination employed. As can be seen all the curves have the general shape shown. The local heat transfer coefficient decreases gradually from the leading edge until point a. This represents the laminar region x_1. Between point a and point b there is a transition region x_2. Beyond point b the local heat transfer coefficient is constant indicating turbulent region x_3.

With the increase of plate inclination turbulence begins earlier and point b is shifted towards the leading edge of the plate resulting in a shorter laminar region and also in a shorter transition region. It is of interest to note that whereas the laminar heat transfer coefficient decreases with the increase of inclination angle θ the turbulent heat transfer coefficient increases.

Transition region

In Fig. 5 are plotted the values of $(Gr_{loc}\,Pr)_{c_1}$ corresponding to points a and b at the beginning and end of transition of Fig. 4. These values may be correlated employing the same form of equation suggested by Vliet [4] for the constant flux condition. The beginning and end of transition of the present data are respectively correlated by the following equations :

$$(Gr_{loc}\,Pr)_{c_1} = 6.3 \times 10^8/e^{4.95\theta} \qquad (1)$$

$$(Gr_{loc}\,Pr)_{c_2} = 1.6 \times 10^9/e^{3.6\theta} \qquad (2)$$

where the inclination angle θ is expressed in radians.

Variation of Nu_{loc} with $Gr_{loc}\,Pr$ in the laminar region

All the laminar region local heat transfer results for the different inclination angles employed could be correlated by

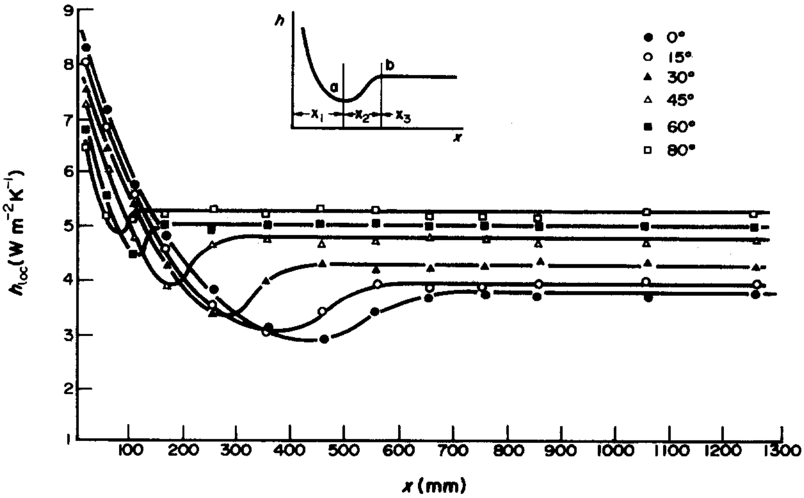

FIG. 4.

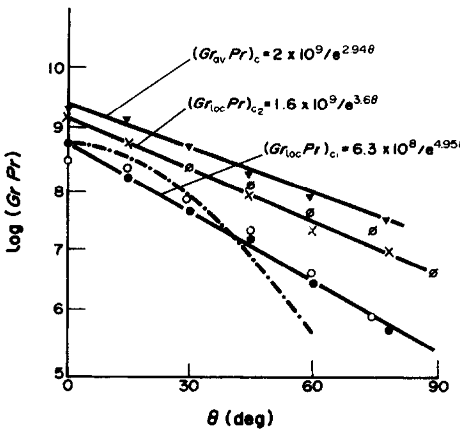

FIG. 5. Correlation of transition data: ○, $(Gr_{loc}\,Pr)_{c_1}$, Hassan and Mohamad [3]; ∅, $(Gr_{loc}\,Pr)_{c_2}$, Hassan and Mohamad [3]; –·–·–, Lloyd and Sparrow [7]; ●, $(Gr_{loc}\,Pr)_{c_1}$, present work; ×, $(Gr_{loc}\,Pr)_{c_2}$, present work; ▼, $(Gr_{av}\,Pr)_c$, present work

$$Nu_{loc} = 0.39(Gr_{loc}\,Pr\cos\theta)^{1/4}$$

$$[2\times 10^4 < Gr_{loc}\,Pr < (Gr_{loc}\,Pr)_{c_1};\quad 0° \leqslant \theta \leqslant 80°]. \quad (3)$$

As can be seen from Fig. 6(a) the deviation of the experimental points from equation (3) is $\pm 11\%$.

Equation (3) confirms the findings of Kierkus [2] and Fujii and Imura [5] that the local heat transfer in the laminar region for any angle of inclination can be represented by a relation between Nu_{loc} and a modified $Gr_{loc}\,Pr$ based on the tangential component of the gravitational force.

Variation of Nu_{loc} with $Gr_{loc}\,Pr$ in the turbulent region

All the turbulent local results for the different inclination angles employed could be correlated by equation (4) within $\pm 14\%$ as can be seen from Fig. 6(b)

$$Nu_{loc} = (0.1+0.05\theta/\pi)(Gr_{loc}\,Pr)^{1/3}$$

$$[(Gr_{loc}\,Pr)_{c_1} < Gr_{loc}\,Pr < 10^{10};\quad 0° \leqslant \theta \leqslant 80°] \quad (4)$$

where θ is expressed in radians.

It may be noted that whereas the effect of increasing θ in the laminar region is to decrease the heat transfer from $\theta = 0°$ to 80° by some 35% the turbulent heat transfer increases in the same range by some 28%.

AVERAGE HEAT TRANSFER

Variation of average heat transfer coefficient with plate length

The sum of the condensed steam from two or more successive compartments of the central zone of the test plate was used to calculate the average heat transfer coefficients h_{av}. Figure 7 shows the variation of h_{av} with the plate length for the inclination angles employed. For all inclinations h_{av} decreases gradually with the increase of plate length until it attains a practically constant value indicating the end of the laminar region. The length of this laminar region decreases with an increase in plate inclination. It is also clear that the average heat transfer coefficient decreases with plate inclination in the laminar region and increases in the turbulent region.

End of laminar region

Whereas a transition region based on h_{loc} exists no transition region based on h_{av} can be detected. Only one critical point representing the end of the laminar region can be read from the curves. As can be seen from Fig. 4 equation (5) approximately represents this critical point for the angles of inclination employed

$$(Gr_{av}\,Pr)_c = 2\times 10^9/e^{2.94\theta} \quad (5)$$

where θ is expressed in radians.

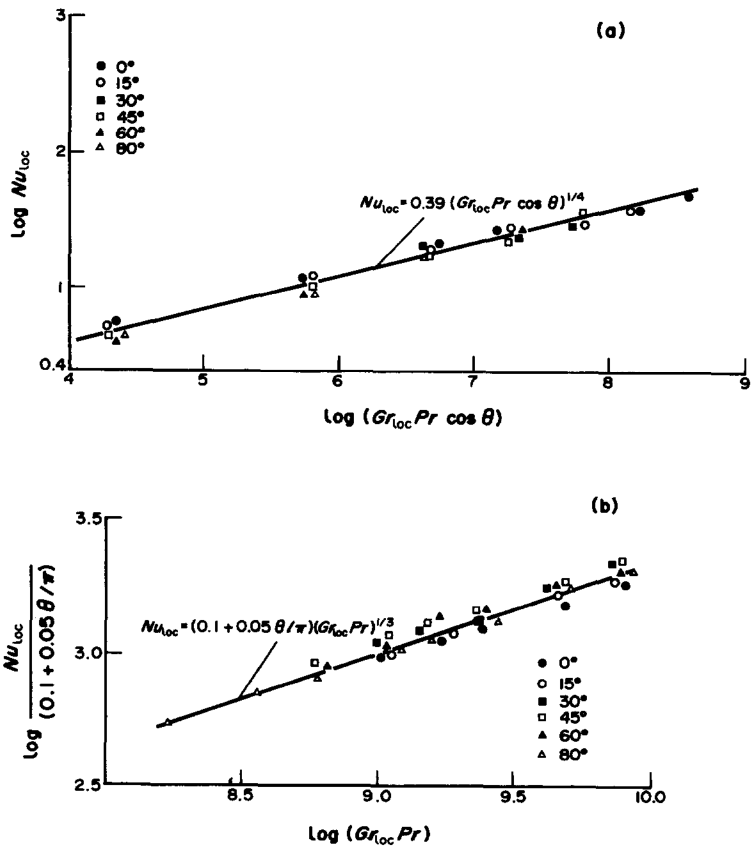

FIG. 6.

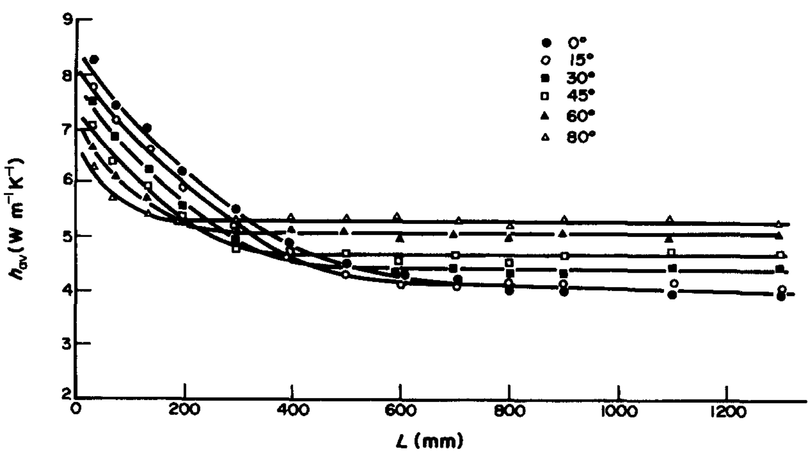

FIG. 7.

Correlation of average heat transfer

The laminar heat transfer data can be correlated by

$$Nu_{av} = 0.54(Gr_{av}Pr\cos\theta)^{1/4}$$

$$[1.15\times10^5 \leqslant Gr_{av}Pr\cos\theta \leqslant (Gr_{av}Pr)_c;\quad 0° \leqslant \theta \leqslant 80°]. \quad (6)$$

Equation (6) represents the general equation for the average natural convection in the laminar region in the range of the present work within $\pm12\%$.

The turbulent heat transfer data can be correlated by

$$Nu_{av} = (0.1+0.05\theta/\pi)(Gr_{av}Pr)^{1/3}$$

$$[(Gr_{av}Pr)_c \leqslant Gr_{av}Pr \leqslant 10^{10};\quad 0° \leqslant \theta \leqslant 80°]. \quad (7)$$

Equation (7) represents the general equation for the average natural convection in the turbulent region in the range of the present work within $\pm12\%$.

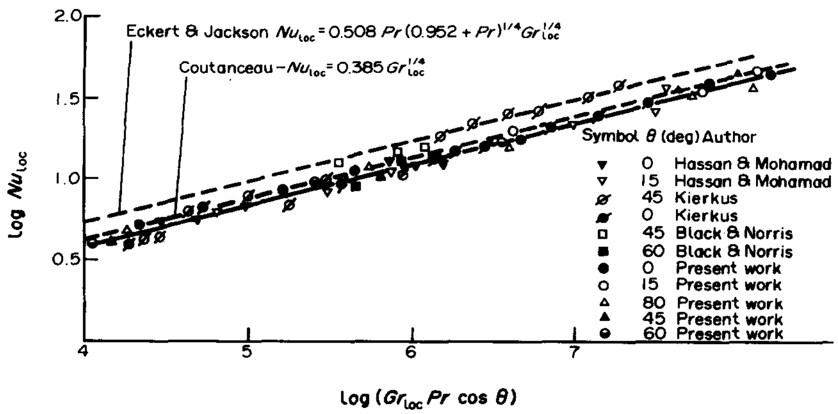

FIG. 8. Comparison with observers' data for local heat transfer in the laminar region.

It will be noted that equation (7) is identical with equation (4) obtained for the local turbulent heat transfer. This can be explained by the fact that the effect of the laminar region on the average turbulent heat transfer is small especially in the higher inclination positions.

COMPARISON WITH PREVIOUS WORK

The present data were obtained using an isothermal plate and will therefore be compared only with previous data obtained under the same heating conditions.

Local laminar heat transfer

Figure 8 shows a comparison of the present data in the laminar region with the experimental results of Kierkus [2], Hassan and Mohamad [3], Black and Norris [6] and the theoretical work of Eckert and Jackson [9] and Coutanceau [10] for the vertical plate. The present data agree very well with the experiments of Hassan and Mohamad [3], with the experiments of Kierkus for $\theta = 0°$ in the whole range of $Gr_{loc}Pr$ employed and for $\theta = 45°$ till $Gr_{loc}Pr = 10^6$. For $\theta = 45°$ and $Gr_{loc}Pr < 10^6$ Kierkus' data are some 25% higher. The experiments of Black and Norris [6] and the theory of Coutanceau [10] agree within $\pm 15\%$ with the present data, whereas the equation of Eckert and Jackson [9] for $Pr = 0.7$ gives as much as 30% higher heat transfer.

It may be noted that the present data are generally lower than most previous data which may be attributed to the isolation of side-edge effects in the present experiments.

Local transition data

On Fig. 5 the experimental values of the local transition region as taken from the local heat transfer data of Hassan and Mohamad [3] are also shown. Lloyd and Sparrow [7] obtained the $Gr_{loc}Pr$ corresponding to the end of the laminar boundary layer along an isothermal inclined plate, which may be considered as the beginning of transition. These data are represented by the dashed line. The data of Hassan and Mohamad [3] show good agreement in the whole range with the present data. The data of Lloyd and Sparrow [7] agree within $\pm 25\%$ with the present data for $0° \leqslant \theta \leqslant 45°$. For $\theta = 45°$ Lloyd and Sparrow's [7] curve gives lower values.

Local turbulent heat transfer

Figure 9 shows a comparison of the present data in the turbulent region with the experimental data for the inclined plates of Rich [1], Hassan and Mohamad [3], Black and Norris [6] and the experimental data for the vertical plates of Coutanceau [10]. Included also is the theoretical equation of Bayley [11] for the vertical position.

All the previous data except those of Black and Norris [6] show good agreement with the present work. This agreement may be explained by the fact that in this region side-edge effects do not affect the local heat transfer in the same way as in the laminar region.

Average heat transfer

To the authors' knowledge the only data available for inclined plates are those of Fujii and Imura [5] for laminar and turbulent flow and Hassan and Mohamad [3] for laminar flow. The data of Fujii and Imura [5] were obtained under conditions different from the isothermal conditions employed by the authors. Only the correlations of Hassan and Mohamad [3] were obtained using isothermal conditions. Their results agree with the present data to within $\pm 6\%$ for the values of θ from 0° to 60°. For $\theta = 75°$, however, Hassan and Mohamad's data are some 23% higher.

CONCLUSIONS

An apparatus permitting the isolation of side-edge effects was used to determine local and average natural convection heat transfer from vertical and inclined plates in the range of $2 \times 10^4 \leqslant Gr_{loc}Pr \leqslant 10^{10}$ and $0° \leqslant \theta \leqslant 80°$. It was found that:

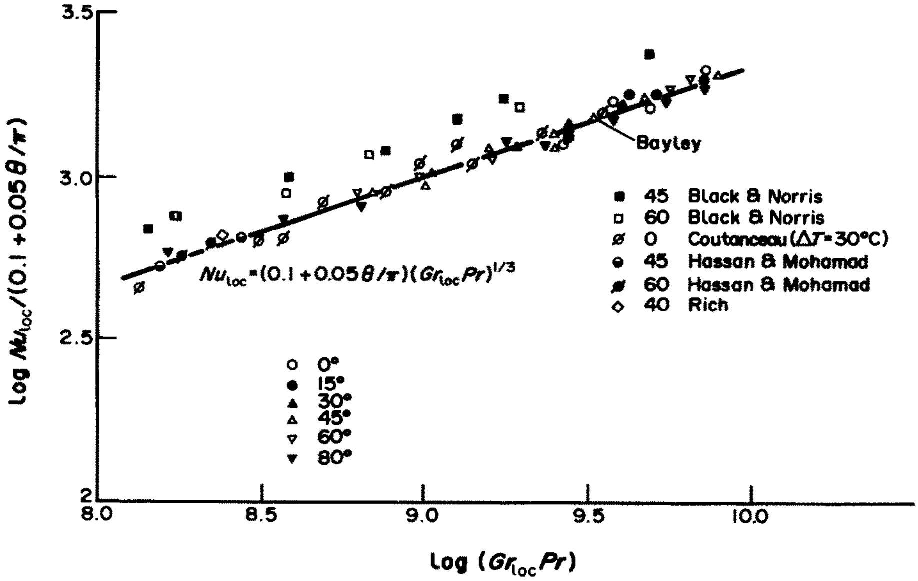

FIG. 9.

(1) Side-edge effects cause high heat transfer near the edges.

(2) For a plate of infinite width (a very wide plate) the use of the tangential component of the Grashof number was successful in correlating the laminar heat transfer data (local and average).

(3) Turbulent heat transfer data could not, however, be correlated by the tangential component of the Grashof number. A different form of correlation was suggested.

(4) Equations are suggested to estimate the beginning and end of the local heat transfer transition regions. For the average heat transfer only one transition point can be detected.

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CONVECTION THERMIQUE NATURELLE SUR DES PLAQUES ISOTHERMES INCLINEES

Résumé—Des données expérimentales locales et globales sur le transfert thermique sont présentées pour la convection thermique naturelle sur des plaques isothermes verticales et inclinées tournées vers le haut dans l'air. Le montage employé permet l'isolement de l'effet de bord, assurant ainsi la condition d'une étendue infinie de plaque. Les expériences couvrent les régions de mouvement laminaire et turbulent. On propose des équations représentant les résultats.

NATÜRLICHE KONVEKTION AN GENEIGTEN, ISOTHERMEN PLATTEN

Zusammenfassung—Es wurden die örtlichen und mittleren Wärmeübergangskoeffizienten bei natürlicher Konvektion an senkrechten und geneigten (Oberseite), isothermen Platten untersucht. Die verwendete Apparatur ermöglicht es, seitliche Randeffekte auszugrenzen, so daß die Situation einer unendlich breiten Platte gegeben ist. Die Experimente umfassen den laminaren und turbulenten Strömungsbereich. Es werden Ausgleichsfunktionen der Meßergebnisse vorgeschlagen.

ТЕПЛОПЕРЕНОС ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ ОТ НАКЛОННЫХ ИЗОТЕРМИЧЕСКИХ ПЛАСТИН

Аннотация—Получены экспериментальные данные для местных и средних коэффициентов теплоотдачи естественной конвекцией от изотермических пластин, расположенных в воздухе вертикально и с наклоном наружной поверхности. В используемой установке исключалось влияние боковых кромок, что давало возможность считать ширину пластин бесконечной. В экспериментах исследовались как ламинарные, так и турбулентные режимы. Предложены уравнения, описывающие полученные результаты.